MODELLING URBAN GROWTH USING CELLULAR AUTOMATA. APPLICATION to the CITY of TANDIL (ARGENTINA)

Linares Santiago*, Juan Pablo Celemín*, Mariana del Fresno** y Meliendrez Martín†

* Centro de Investigaciones Geográficas, IGEHCS - CONICET / UNCPBA, Argentina
** PLADEMA/UNCPBA – CIC-PBA, Argentina
† Departamento de Computación y Sistemas, UNCPBA, Argentina

Abstract

It is evident in studies of processes taking place in urban spaces that the modeling of emergent phenomena has established itself as an acceptable methodology to address complex processes that characterize socio-spatial dynamics. Most of the background on this approach has successfully applied cellular automata to simulate processes such as automotive and pedestrian movements, land use changes, emergence of new urban centers, urban expansion and socio-spatial segregation. However, a relatively small progress has been made on this subject in the Latin American urban space as well as in the developed of computer applications that models emerging simulation. In this context, the main the objective of this work is to build a computer application that facilitates the study and understanding of urban growth and expansion phenomena in Argentinean cities of intermediate size by using cellular automata.

Address for correspondence: Santiago Linares, Centro de Investigaciones Geográficas, IGEHCS - CONICET / UNCPBA, Tandil - Buenos Aires - Argentina, B7000GHG. E-mail: slinares@fch.unicen.edu.ar

Acknowledgements: we would like to thank Cristina Beatriz Massera, at the National University of Patagonia San Juan Bosco, for sharing the software license to develop this application.

1 Introduction

The theoretical and methodological framework underlying traditional models of urban structure refers to a historical period in which the city was very different according to contemporary manifestations. Traditionally, cities were represented with a single central district characterized by the concentration of raw materials, labor, and density of shops and services; while at present this scenario is substantially transformed as cities tend to be polycentric due to massive automobile access, the spread and diversification of services and new information and communication technologies.

This is why many current approaches, based on classical theories of urban development, find limitations to simulate it and describe the changing nature of sub-spaces that are developed inside such cities. Consequently, there is a need to implement models that are as flexible and dynamic in their simulation capabilities as the city in its ability to evolve.

It is possible to find a viable solution to these problems in complexity theory-based models. This approach is based on delegating the simulation at macro-scale urban structure to a set of dynamic sub-models at micro-scale derived from the complexity theory. These approaches try to represent individual actors (or groups) in a given system that can interact with each other and/or with an environment. The macro-level behaviors are configured from the aggregation of these interactions.

Complexity theory can be considered as a new systemic approach studying the relationship between the parts and the whole in a different way, emphasizing the idea of an emerging structure from a bottom-up process, where local actions and interactions produce the overall pattern. The intra-urban dynamic processes are more important than the structure itself, allowing the understanding of such systems to go beyond the description (in static terms), to capture the inner essence of the phenomena of change (Batty 2000, Wu 2002, Casti 1997).

Different models from complexity theory have been used to simulate urban dynamics, within which are statistical models, agent-based models, fractal models, models based on cellular automata models and artificial neural network models (Batty 2005). In this paper we present a software application that takes advantage of the developed models based on cellular automata rules.

2 Cellular automata applied to urban growth

The seminal work where the concept of cellular automata (CA) to address socio-spatial processes took place in 1979 when the geographer Waldo Tobler produced the concept of CA to model and predict urban growth in a city (Tobler 1979). In his paper he proposed the idea of how CA can function as a useful tool for urban planning and how it can get the best transition rules. Tobler defined a law that would be very important in the development of predictive models for urban growth: “In geography everything is related to everything else, but near things are more related than distant things”. This means that the proximity or distance between certain types of processes or activities inhibits or stimulates the emergence and development of other activities nearby.

Many authors followed the line of the model defined by Tobler, such as Couclelis (1985, 1988), Itami (1988) and Phipps (1989) but it was only from the 1990s on when models allowed and accurate representation of reality.

One of the most important methodological proposals is the SLEUTH model developed by Clarke et al. (1997), which uses five variables as input defined as follows: *Slope*, *Land use area*, *Excluded area*, *Urban area*, *Transportation map*, *Hillshade area*. At the same time, the model uses five factors to control the system behavior: diffusion, reproduction, spread, slope and distance to routes.

CA theory assumes that the potential of a cell to undergo some transformation of land use depends on the states of neighboring cells. White and Engelen's cellular automata (1993) consist of a finite cellular space representing a hypothetical urban area. The different land uses are identified with elements of the set of states of CA. Two types of states are defined: active and fixed. The first represents conventional land uses such as residential or commercial, which may change over time. The second represents road infrastructure or natural features such as a river or a canyon.

State transition rules are defined through a function that relates four different factors:

- The intrinsic convenience between different land uses, representing heterogeneous aspects of the geographic space that is being modeled. These conveniences for land use, located at a specific point, transformed into another or remaining unchanged are related to issues ranging from soil quality to legal restrictions or speculative economic pressures.
- The effect on a specific land use with surrounding land uses. This type of effect can be attractive or repulsive, as some land uses attract some and repel others. For example, a residential land use attracts business while repelling industries.
- The effect of local accessibility, which represents the ease of access to the transport network.

- The stochastic disturbance that captures the effect of imperfect knowledge and variable behavior of social actors in relation to land use.

The model of White and Engelen introduces a set of changes from classic CA models as it differentiated from the use of monotonous decay of the influence of the neighborhood as distance increases. Instead, it used $W_{x,d,j}$ weights to represent the balance of the forces of attraction and repulsion that occur in the different types of land use. As the sum of two opposing forces, the $W_{x,d,j}$ weight is not necessarily monotone and may even be negative. It also introduces a strict order of possible transitions between land uses. The model only allows land use change following a pre-defined sequence. For example, given a model where there are 4 possible states of a possible land use, change sequence is: free → dwelling → industrial → commercial.

More recently, this strict order of possible transitions pre-defined by the user has been in fact superseded by fuzzy rankings of transitions. According to such type of ranking, not always the cells with the highest potential to undergo changes in land use will actually suffer this change, since transitions are subject to some degree of randomness, so as to render the simulated urban environment as close as possible to reality.

3 Methodological details of the proposed model

Many studies have shown that the integration of Geographic Information Systems (GIS) and CA models can be used to understand, simulate and predict urban growth successfully. In this framework we develop an application in Java language, called

SACcity, which uses the functionality and data model from ESRI company products, such as ArcGIS Desktop 9.3 and ArcGIS Engine 9.3.

Developing applications using the features developed by ESRI has a number of advantages:

- The framework provided by ArcGIS for developing GIS applications, is the same used in the construction of the ArcGIS Desktop products, making it easier for inputs and outputs of the developed program to be manipulated with other existing ArcGIS products.
- To facilitate applications design, ArcGIS Engine includes a set of reusable components to provide graphics functions, which are developed as AWT components. The Java API for ArcObjects allows these controls to be available for use as visual components.
- ArcGIS Engine provides multiple interfaces for programming interfaces (APIs). The ArcGIS Engine Development Kit software for Java comes with a collection of tools to perform a number of functions such as: display maps with multiple layers, search and find features on the map, draw images from aerial photographs or satellite images, draw graphic features such as points, lines, circles and polygons and identify features on the map.

Figure 1 shows user interface where integration of both components can be observed.

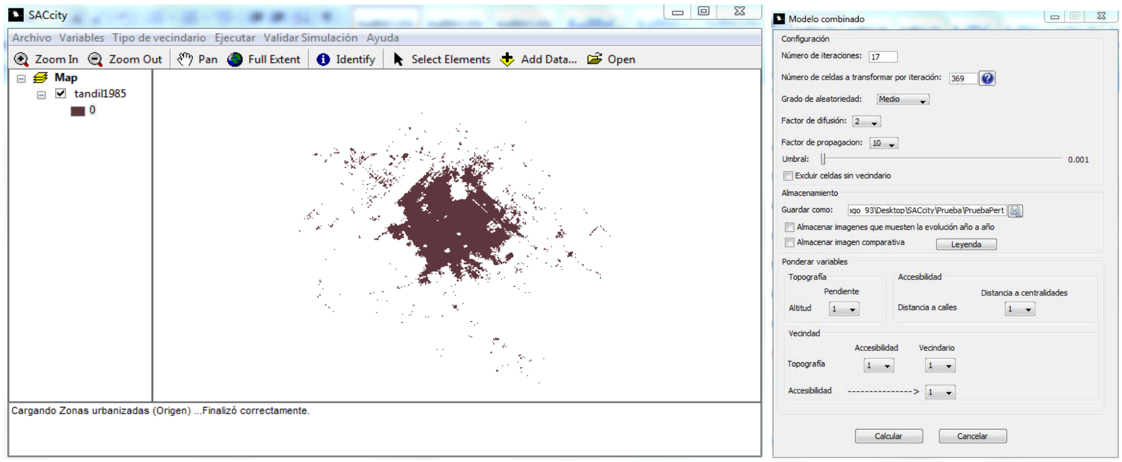
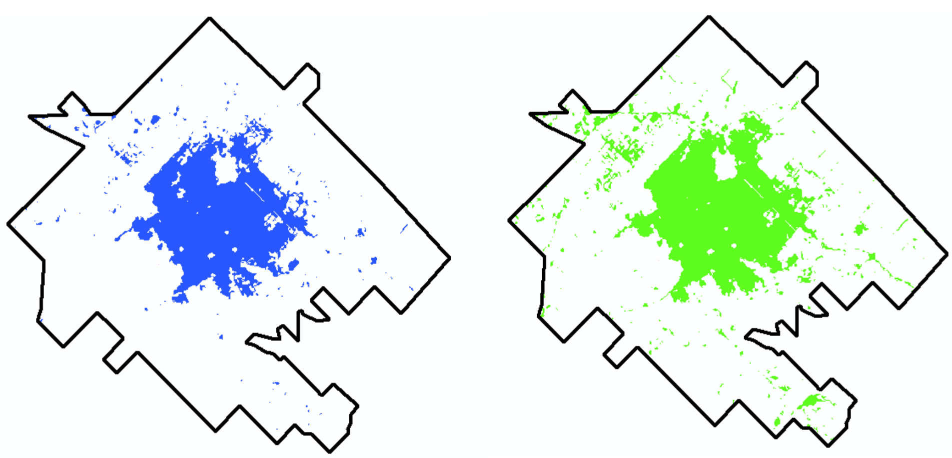


Figure 1 SACcity Application

3.1 Model Variables

To run the simulation model the user must enter a set of thematic layers representing different factors affecting (to a greater or lesser extent) urban growth. The variables considered in this application are the usual present in CA research.

First, we took into account the urbanized land represented by the urban area built in the city of Tandil in 1985 and 2002 (Figure 2), which is the variable to be explained or dependent variable.



(a) (b)

Figure 2 Map of built-up areas: Tandil, 1985 (a) and 2002 (b)

Transport network plays a key role in the process of urban expansion, and that is why variables of distance to roads were taken into account. Distance is simply calculated as the Euclidean distance to the nearest road of transport and is considered a measure of geographical accessibility, so the lowest distance values imply a greater likelihood of change. Figure 3 shows a map of distances to roads represented with a gray scale, ranging from the black (zero distance, or minimum distance that can exist between a point in the study area and the nearest street to that point) gradually varying in intensity until it reaches the white color (maximum distance that can exist between a point in the study area and the nearest street to that point).

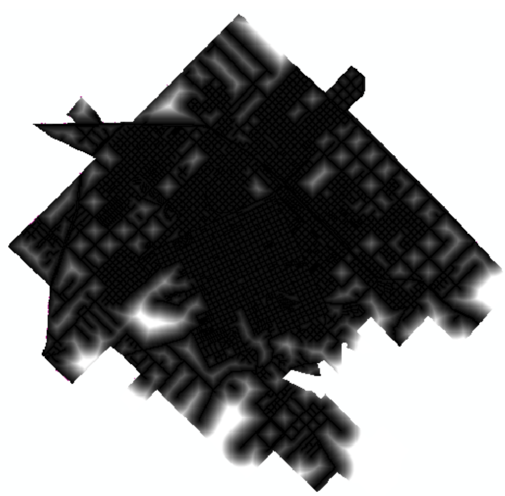


Figure 3 Map of distances to streets

Finally, we take into account the variables of slope and altitude (above sea level). In this case the slope is variable between 0% (0 degrees) and 100% (90 degrees) while the height is a variable which can take any positive value. The greater the angle of the slope, the lower the likelihood of change. Similarly, and in accordance to building restrictions that characterize Tandil, the lower the altitude, the greater the likelihood of change. Figure 4 shows maps of slopes (a) and altitudes (b), also represented with gray scales. For the slope map, black equals the smallest degree of slope that exists in the study area, while white represents the highest possible degree. For the map of altitudes, black symbolizes the lowest altitude that exists in the study area, while white symbolizes the maximum altitude.

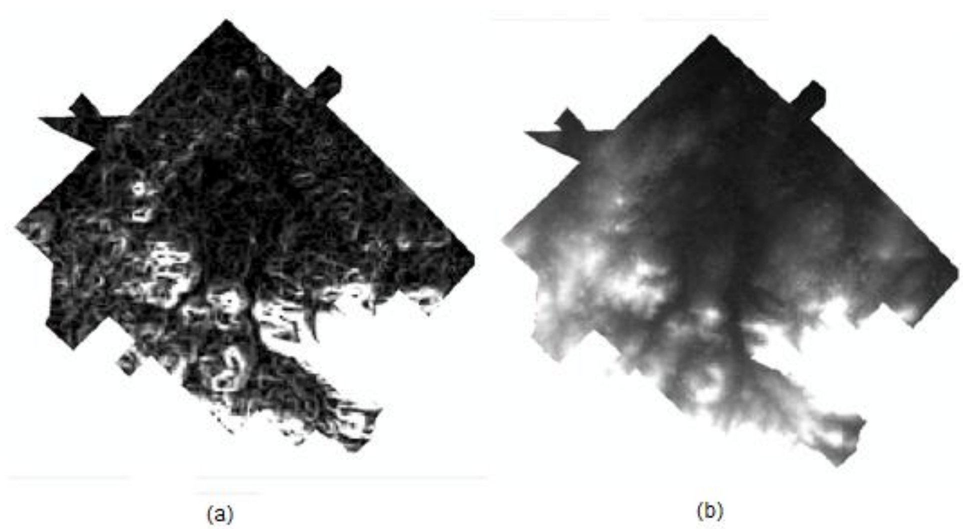
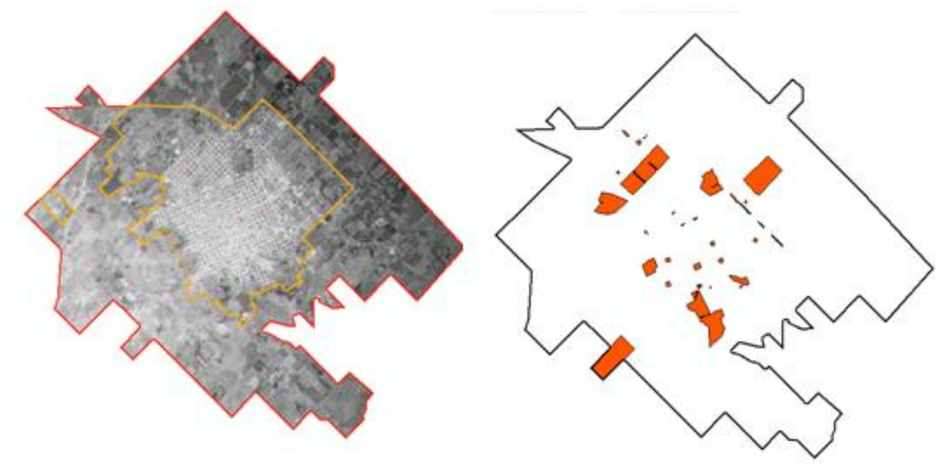


Figure 4 Map of slopes (a) and elevation map (b)

Planning policies also determine urban processes and therefore two basic aspects associated with them have been included. First, by defining the potential area to develop, according to the Spatial Development Plan of Tandil (2005); and secondly by including an analysis mask that eliminates areas with restrictions for future development such military, recreational or community facilities sectors (Figure 5).



(a) (b)

Figure 5 Map of complementary and urban areas (a) and restrictive areas (b)

3.2 Modeling algorithms to predict urban growth

The built application allows the execution of two different models: a **classical CA model**, which from simple local rules strives to generate complex patterns complex patterns of urbanization globally; and a **combined CA model**, which incorporates a multi-criteria decision method, introducing a diffusion factor and where the transition rule is defined as a function that includes all input parameters in combination, thus getting closer to a real simulation.

The CA model distinguishes between two land uses, defined according to the classification system of Anderson et al. (1976) as urbanized and non-urbanized. The urbanized land use includes residential, commercial and service land, industrial, transport and communication, community facilities or mixed uses. The remaining land uses are grouped into one category as non-urbanized, where forests, pastures, vacant land or crops are included.

For the CA model we adapted the one proposed by White and Engelen (1997), introducing diffusion and propagation factors according to Clarke et al. (1997) and adding a multi criteria decision method for weighing variables. The final model includes various components present in cellular automata models:

- **Finite set of states**: unlike the model developed by White and Engelen, where various types of land use are considered, each cell may belong to the category of *urbanized* or *non-urbanized*. Only the transition from *non-urbanized* to *urbanized* is allowed.
- **Neighbourhood**: There is a matrix of 9 x 9 cells resulting in 81 neighboring cells.
- **Evolution rule**: the transition potential is calculated for each automata cell and at the end of each iteration the cells with the greatest potential are transformed to the *urbanized* state. For those cells whose state is already *urbanized*, transition potential is not calculated since its state cannot be modified.
- **Calculation virtual clock:** each iteration of the model involves an application of the evolution rule. These iterations represent a unit of time of growth.

The function to calculate the transition potential from the current state of a *C* cell to the *i* state is calculated as follows:

$$P_{C,i} = I_{C,i} * \vartheta^{1} + A_{C,i} * \vartheta^{2} + N_{C,i} * \vartheta^{3} + \varepsilon \quad (1)$$

where:

- $P_{C,i}$ is the *C* cell potential for its land use becomes *i*.

- ε is a random disturbance term. Since in the formula used in the present model the values of the potential of each cell range from 0 or 1, four possible values for adjusting the degree of randomness where defined: null, where the perturbation term is not used to calculate the potential of the cells; low, values between 0 and 0.1; average, values between 0 and 0.3; and high, values between 0 and 0.6.
- $I_{C,i}$ is the effect that the topography has on the *C* cell for it to have a land use *i*. It is calculated as:

$$I_{C,i} = Slope * \vartheta^{4} + Altitude * \vartheta^{5} \tag{2}$$

where

$$Slope = \frac{MaximumSlope - PointSlope}{MaximumSlope} \tag{3}$$

Altitude calculation is similar to that shown for *Slope*.

- $A_{C,i}$ is the effect that accessibility has on *C* cell for it to have a land use *i*.

$$A_{C,i} = DistanceCenter * \vartheta^{6} + DistanceStreet * \vartheta^{7} \tag{4}$$

- *DistanceCenter* and *DistanceStreet* are calculated similarly to Equation 3.

$N_{C,i}$ is the effect that the neighborhood has over the *C* cell for it to have a land use *i*. To determine the effect that a cell in the neighborhood has over the central cell it was decided to use a Radio 4 Moore neighborhood, meaning that the value of the effect in the neighborhood of a cell depends on the 80 cells surrounding it (see Table 1). This effect of decay indicates that the influence of "attraction" of urban areas on the neighboring cells when new buildings are developed becomes smaller as we move away from them. This same effect has been used by other models based on cellular automata (White et al. 1997;

White and Engelen 2000; Barredo et al. 2003).

Table 1 Matrix showing the effect of neighboring cells over the central cell.

0	0.15	0.3	0.6	1	0.6	0.3	0.15	0
0.15	0.3	0.6	1.25	2	1.25	0.6	0.3	0.15
0.3	0.6	1.25	3.5	5	3.5	1.25	0.6	0.3
0.6	1.25	3.5	7	10	7	3.5	1.25	0.6
1	2	5	10	1	10	5	2	1
0.6	1.25	3.5	7	10	7	3.5	1.25	0.6
0.3	0.6	1.25	3.5	5	3.5	1.25	0.6	0.3
0.15	0.3	0.6	1.25	2	1.25	0.6	0.3	0.15
0	0.15	0.3	0.6	1	0.6	0.3	0.15	0

- Introducing the $\vartheta^1, \vartheta^2, \vartheta^3, \vartheta^4, \vartheta^5, \vartheta^6, \vartheta^7$ weighing parameters is a major requirement of the model as it allows to modify the importance of input variables performing the simulation according to the Analytic Hierarchy Process (Saaty 1980).

Two interesting aspects in the SLEUTH model are the diffusion factor and the propagation factor that limits the concentration of cells that transited form non-urbanized to urbanized in one area after a certain number of iterations.

The diffusion factor values fall in the range between 0 and 4 and is used when the state of a cell is about to be transformed. For example a factor equal to three indicates that the cell can only be passed to "Urbanized" if, and only if, within a radius of 3x3 no cell has been transformed in the same iteration.

The propagation factor is used to control the percentage of cells that are passed to urbanized state in proximity or remoteness to the existing buildings. Values range from 0, where all the cells are chosen to transform starting with those which are closest to urbanized cells, to 10 which means that 100% of the cells are selected to be transformed over those further away from urbanized cells. To define the concepts of closeness or distance in the model, a neighborhood of 80 cells was used surrounding the concerning cell; thus, nearby cells are those whose neighborhood value is high, and distant cells are those whose neighborhood value is minimal. The cells with values below the threshold set by the user are excluded from the computations. This threshold expresses standardized scores between 0 and 0.7, where 0 indicates the lowest possible neighborhood and 0.7 indicates close proximity to the maximum, that is 1.

Finally the models includes different procedures to validate the result, that is, to check what is the degree of correspondence between the result predicted by the model and the reality. To this end, two methods are used in this SACcity: *visual comparison of the maps* (1) and a *quantitative evaluation using confusion matrixes* (2). The confusion matrix shows four accuracy measures: *overall accuracy*, *user accuracy*, *producer accuracy* and also the *Kappa coefficient* which has received critics as a comparison method in this type of analysis (Pontius and Millones, M., 2011) but it is still present in various CA models.

4 Application and analysis of results

Tandil is the main city of the homonymous department, located in Argentina, in the southwest of Buenos Aires Province. Located at 331 km from Buenos Aires, the federal capital. It has a population of 116,945 inhabitants according the latest census (2010). The urban area covers 48 km^2 with an average population density of 2,436 inhab./ km^2 . Like in most Latin American cities, the population decreases from the center to the periphery of the town.

To obtain an optimal simulation of the city of Tandil during the period considered (1985-2002) it was necessary to make an assessment of the impact or sensitivity that the parameters included in SACcity have over the growth and expansion of the city.

The first parameter analyzed was the optimal degree of randomness that simulates the expansion of the city of Tandil. It admits four levels (null, low, medium and high), capturing the imperfect knowledge and the changing trends among the actors involved in the production and structuring of urban space.

Figure 6 shows how the inclusion randomness increasingly favors the simulation results, corroborating the existence of random factors in territorial processes that occur in reality. We can also say that the average level seems to be the most appropriate to be implemented when simulating the expansion of cities, or at least in midsize cities.

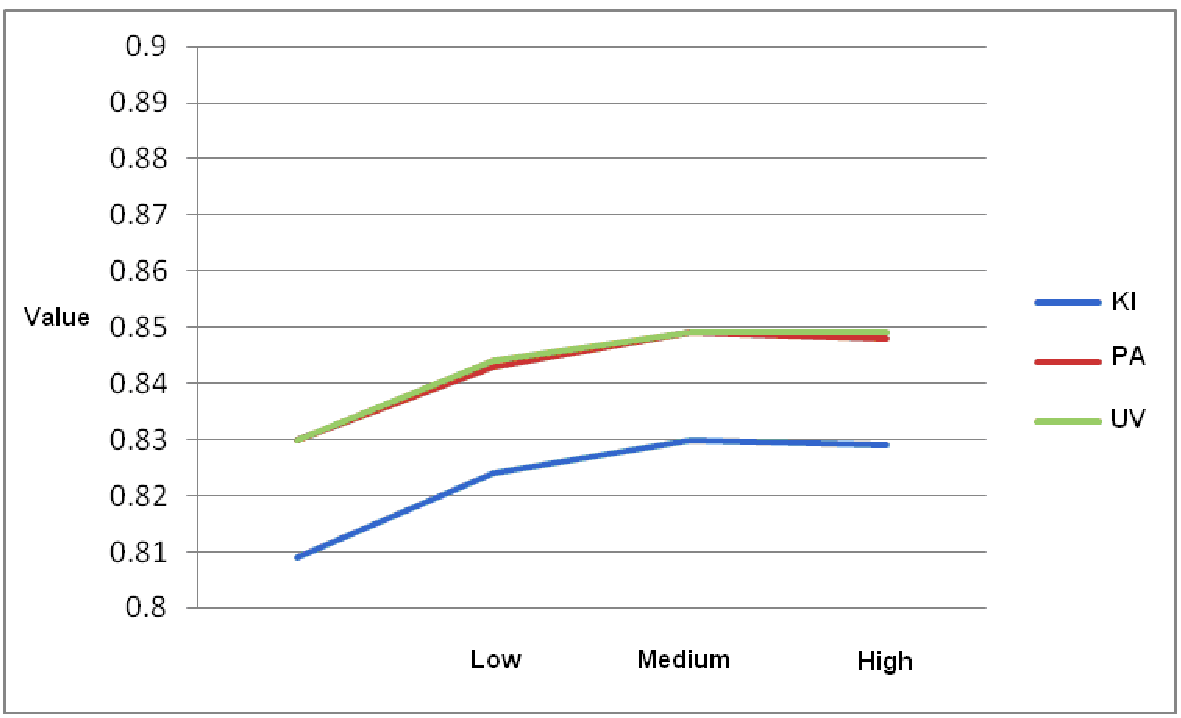


Figure 6 Performance of validation index according to the degree of randomness introduced to the model

Following the requirements to run the combined CA model it is necessary to complete the diffusion and propagation factors. Both, though with different features, allow the user to enter the degree of dispersion associated with the urban growth of the city and can be incorporated in the subsequent simulation. Figure 7 displays the results of running the model with different values of diffusion and propagation. This parameter evaluates whether the growth of a city occurs continuously or compact, or conversely and if it shows a scattered or diffuse expansion. For Tandil high and medium-high values of propagation would exaggerate urban sprawl, while null values are associated with extreme compactness and do not seem appropriate either. Therefore medium values for diffusion and propagation would be the most representative for this scale of urban analysis.

Evaluation			**Parameters**	**Configuration**
KI	PA	UV		
			diffusion	1
0.859	0.866	0.885	propagation	1
			threshold	0.7
			diffusion	2
0.864	0.863	0.895	propagation	4
			threshold	0.5
			diffusion	3
0.813	0.834	0.834	propagation	7
			threshold	0.2
			diffusion	4
0.752	0.78	0.78	propagation	10
			threshold	0.05

Figure 7 Validation indices according to diffusion and propagation factors introduced to the model

Finally, it is possible to differentially weigh the considered variables for running the model. This weighing, which is based on *Analytic Hierarchy Process* (Saaty 1980) based on the Analytic Hierarchy Process, gives the user the possibility to accordingly assign weights to the considered variables with respect to his/her knowledge of the study area, what is crucial to explain the expansion of the city.

To find the sensitivity that each factor has to explain for the urban expansion of Tandil, we conducted three trials for the factors considered in our research (Topography, Accessibility and Neighbourhood). The result present in Figure 8 show the prominent role that the network of transport configuration has over the expansion of Tandil, while the Accessibility factor, compounded by the distance to streets and roads and distance to the city center, best explains the horizontal growth of the city. Also contiguity relations could be identified whereas the topological characteristics seemed to be a minor obstacle for the construction of new buildings.

Combined CA model	
Result	Parameters

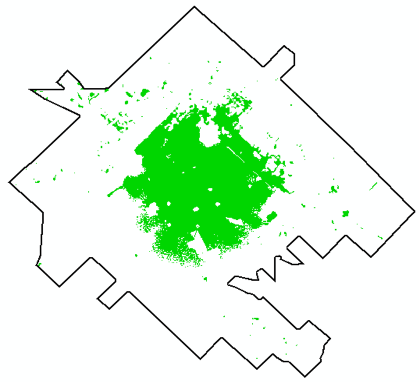
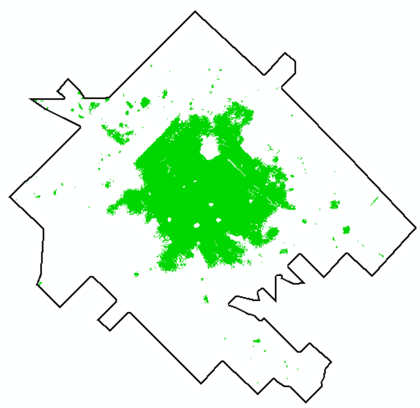
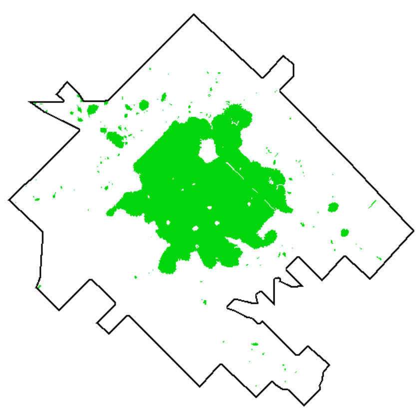
	*Randomness: Medium* *Diffusion: 2* *Propagation: 4* *Threshold: 0.5* *Weighing: T=9 A=1/9 N=1/9* *Validation* *Kappa Index: 0.824* *Producer´s accuracy: 81.3%* *User´s Accuracy: 87.6%*
	*Randomness: Medium* *Diffusion: 2* *Propagation: 4* *Threshold: 0.5* *Weighing: T=1/9 A=1/9 N=9* *Validation* *Kappa Index: 0.866* *Producer´s accuracy: 85.6%* *User´s Accuracy: 90.0%*
	*Randomness: Medium* *Diffusion: 2* *Propagation: 4* *Threshold: 0.5* *Weighing: T=1/9 A=9 N=1/9* *Validation* *Kappa Index: 0.87* *Producer´s accuracy: 86.9%* *User´s Accuracy: 90.0%*

Figure 8 Results of the combined model by varying the factors weights

5 Conclusion and recommendations for future work

There are numerous models that simulate urban growth. In this particular research, we presented SACcity a tool whose concrete contribution to the research on urban modeling is that it combines the usual parameters of randomness, diffusion and propagation parameters in an AHP-calibrated urban CA model based on a set of variables related to infrastructure and physical characteristics. This model is being developed at the Center for Geographical Research, Faculty of Human Sciences at the UNCPBA (Universidad Nacional del Centro de la Provincia de Buenos Aires).

The results of the proposed model corroborate the versatility of the application as it can be adapted to the user´s requirements: the possibility of incorporating randomness, diffusion and propagation parameters in a single application favors the adaptation of the model to different urban contexts. Similarly, differential weighing allows the user to give varied importance to each of the input factors, and as shown in our example, actually to know which of them is crucial to explain how a city expands.

Summarizing these features, we could say the application of CA in the investigation of urban phenomena means narrowing the distance between the processes occurring in reality and those modeled by computer tools. This was demonstrated by the results obtained in the validation of the herein reported experiments. However, there are many areas where progress could be made in this application or in similar future developments.

In principle, the variables considered in this work (urbanized area, distance to transport network, distance to centralities, elevation, slope, developable area and prohibited areas) are the ones regarded as the major drivers of urban growth. Nevertheless, the model at issue could be significantly optimized if other variables related to environmental and institutional aspects of the site were included, such as geotechnical characteristics of the soil, risk areas location, and urban zoning regulations.

Moreover, the models developed in this work only consider two possible types of land use: urbanized and non-urbanized. Thus, the simulations allow us to see how the city grows and expands as a whole. Future studies could incorporate more classes that represent more faithfully the internal morphology of cities.

Finally, a new generation models could be incorporated to this research, such as models based on multi-agents that are showing excellent properties to represent the complexity that characterizes the socio-spatial dynamics (Hatna and Benenson 2012).

Bibliography

Aguilera F 2006 Predicción del crecimiento urbano mediante sistemas de información geográfica y modelos basados en autómatas celulares. *Revista Internacional de Ciencia y Tecnología de la Información Geográfica* N° 6: 81-112

Aguilera F, Soria J A and Valenzuela L M 2006 Explorando el crecimiento en la aglomeración urbana de Granada: un modelo basado en autómatas celulares. In *El acceso a la información espacial y las nuevas tecnologías geográficas: actas [del] XII congreso nacional de tecnologías de la información geográfica*. España: 3-16

Anderson J R, Hardy E E, Roach J T and Witmer R E 1976 A land use and land cover classification system for use with remote sensor data. U.S. Geological Survey Professional Paper N° 964

Barredo J I, Kasanko M, McCormick N and Lavalle C 2003 Modelling dynamic spatial processes: simulation of urban future scenarios through cellular automata. *Landscape and Urban Planning* Vol. 64: 145-160

Batty M, Couclelis H and Eichen M 1997 Urban systems as cellular automata.

Environment and Planning B: Planning and Design. Vol. 24. N° 2: 159-164

Batty M 2000 Less Is More, More Is Different: Complexity, Cities, and Emergence. *Environment and Planning B: Planning and Design*. Vol. 27. N° 2: 167-168

Batty M 2005 *Cities and complexity: Understanding cities with celular autómata, agent-based models, and fractals*. Cambridge, MA: The MIT Press.

Buzai G D 2005 Actualización de cálculos y distribuciones espaciales a través de Cadenas de Markov y Autómatas Celulares: Pérdida de suelos en el Área Metropolitana de Buenos Aires. *Primeras Jornadas Argentinas de Ecología de Paisajes*, GEPAMA/IALE. Buenos Aires. CD ROM.

Casti J L 1997 *Would-Be Worlds: How Simulation Is Changing the Frontiers of Science*. London, John Wiley and Sons

Chuvieco E 2008 *Teledetección ambiental: la observación de la tierra desde el espacio*. Barcelona, Editorial Ariel S. A. 3ª Edición

Clarke K C, Hoppen S and Gaydos L J 1997 A self modifying cellular automanton of historical urbanization in the San Fransico Bay area. *Enviroment and Planning B*. Vol. 24. N° 2: 247-261

Couclelis H 1985 Cellular worlds: a framework for modeling micro-macro dynamics. *Environment and Planning B*. Vol. 17. N° 5: 585-596

Couclelis H 1988 Of mice and men: what rodent populations can teach us about complex spatial dynamics. *Environment and Planning A*. Vol. 20. N° 1: 99-109

Henríquez C, Azócar G and Aguayo M 2006 Cambio de uso del suelo y escorrentía superficial: aplicación de un modelo de simulación espacial en Los Ángeles, VIII región del Biobío, Chile. *Revista de Geografía Norte Grande*: 61-74

Hatna E, Benenson I 2012 The Schelling model of ethnic residential dynamics: beyond the integrated – segregated dichotomy of patterns. J Artif Soc Soc Simul 12(1):6

Itami R 1988 Cellular worlds: models for dynamic conception of landscapes. *Lanscape Architecture* Vol. 78. N° 5: 52-57

Linares S 2015 Aplicación de modelos de simulación de crecimiento urbano. In Buzai, Cacace, Humacata and Lanzelotti (comp.), *Teoría y métodos de la Geografía Cuantitativa. Libro 1: Por una Geografía de lo real*. MCA Libros, Mercedes. Argentina: 231-247.

Phipps M 1989 Dynamic behavior of cellular automata under the constraint of neighborhood coherence. *Geographical Analysis* Vol. 21. N° 3: 197-215

Pontius Jr., R.G., Millones, M., 2011. Death to Kappa: birth of quantity disagreement and allocation disagreement for accuracy assessment. International Journal of Remote Sensing 32 (15), 4407–4429.

Saaty T L 1980 *The Analytic Hierarchy Process*. McGraw-Hill. Nueva York. 1980

Sui D Z and Zeng H 2001 Modeling the dynamics of landscape structure in Asia's emerging desakota regions: a case of study in Shenzhen. *Landscape and Urban Planning*. Vol. 53. N° 1-4: 37-52

Tobler W 1979 Cellular Geography. In Gale S and Olsson G (eds.), *Philosophy in Geography*, London, D. Reidel Publishing Company: 379-386

Von Neumann J 1966 *Theory of self-reproducing automata*. University of Illinois Press, Urbana: 63-87

White R and Engelen G 1993 Cellular-Automata and Fractal Urban Form: a Cellular Modeling Approach to the Evolution of Urban Land-Use Patterns. *Environment* *and Planning A*. Vol. 25. N° 8: 1175-1199

White R 1997 Cities and Cellular Automata. *Discrete Dynamics in Nature and Society* Vol. 2: 111-125

White R and Engelen G 1997 Cellular automata as the basis of integrated dynamic regional modelling. *Environment and Planning B: Planning and Design*. Vol. 24. N° 2: 235-246

White R, Engelen G and Uljee I 1997 The use of constrained cellular automata for high resolution modelling of urban land-use dynamics. *Environment and Planning B: Planning and Design*. Vol. 24. N° 3: 323-343

White R and Engelen G 2000 High-resolution integrated modelling of the spatial dynamics of urban and regional systems. *Computers, Environment and Urban Systems*. Vol. 24. N°5: 383-400

Wolfram S 1986 Theory and applications of cellular automata. *World Scientific Publishing Co*. Hong Kong: 333-342

Wu, F. Calibration of stochastic cellular automata: the application to rural urban land conversions. *International Journal of Geographical Information Science*. Vol. 16. N° 8: 795-818